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A Model of the Firm in Time and Space

Lanny Arvan

Leon N. Moses

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A Model of the Firm in Time and Space

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A Model of the Firm in Time and Space

by Lanny Arvan* and Leon N. Moses**

Abstract

We consider a multiplant monopoly that sells to markets which are geographically separated and which stores product over time via an inventory capability. It is assumed that plant average production cost is U-shaped and that if the output of a plant's production run were sold to a single market at only one point in time the plant would operate on the falling portion of its average cost curve. Hence, it is in the interest of the firm to aggregate markets, both spatially and temporally, to lower average production cost. We develop the optimal joint interplant spacing-inventory policy. We also consider the effects changes in freight costs, storage costs, and interest charges have on the firms optimal policy.

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A Model of the Firm in Time and Space

by

Lanny Arvan* and Leon N. Moses**

I. Introduction

The firm about which we reason in this paper will be familiar to all economists. It is the firm of imperfect competition theory which faces a downward sloping demand function in each of its markets, and whose cost function has the traditional shape. That is, whether due to economies of scale in the long run or variable returns in the short run, its marginal and average cost functions exhibit declining and increasing stages. Our firm differs from the one of traditional theory in that it can only achieve maximum profits by framing a plan in which temporal and spatial factors are taken into account simultaneously. It aggregates markets, operates multiple plants, and ships goods through time and space.

Our model of this firm involves three bodies of literature that have developed apart from one another. They are the theories of the multi-plant firm, of location, and of the dynamics of production-inventory behavior.¹ One goal of the paper is to show the relationships between these three types of studies and thereby to contribute to bringing them and the researchers who work on them closer together.

As was indicated above, our firm has a U-shaped cost curve. We assume that the demand function it faces at any time-space point intersects the plant average cost function at a point on its declining segment. Given this condition, the firm may find it economical to

aggregate the markets it serves, both spatially and temporally. It will find it profitable to aggregate markets spatially if the costs of carrying goods through space are not too great relative to the production cost economies that can be achieved by concentrating production, and if the overall production costs of the firm's operations are not related to the number of plants it manages. In this situation the firm will concentrate production, but not completely. It will operate multiple plants and bear the cost of transporting its product to consumers who are located away from a plant. Similarly, if the costs of carrying goods through time are not too great relative to the economies of concentrating production in time, and if there are no start-up costs after a plant has been down for a time, the firm will also behave dynamically, producing today for sale in future days. Moreover, it will behave in this way even though its demand and cost functions are known with certainty and are temporally invariant. If the firm's average cost function rose throughout it would behave differently. That is, it would establish a plant at each geographic market point and avoid paying transportation fees. It would also behave statically, avoiding storage and interest costs on inventory by satisfying the demands of each period strictly from output of that period, which is the temporal analog of having a plant at each geographic market point.

Even with temporal and spatial aggregation of markets, our firm operates each of its plants to the left of minimum average cost. The profit maximizing plan it develops involves decisions as to: the number, spatial distribution, and output of plants over time; the geographic extent of the market served by each plant; the different

(discriminatory) prices that consumers pay and the quantities they purchase, depending on their locations relative to the location of the plant that serves them; the duration of the temporal market satisfied from the output of each production run; the different (discriminatory) prices consumers pay and the quantities they purchase, depending on their temporal location relative to the time the output they are buying was produced; and the stock of finished product inventory carried at each point of time.

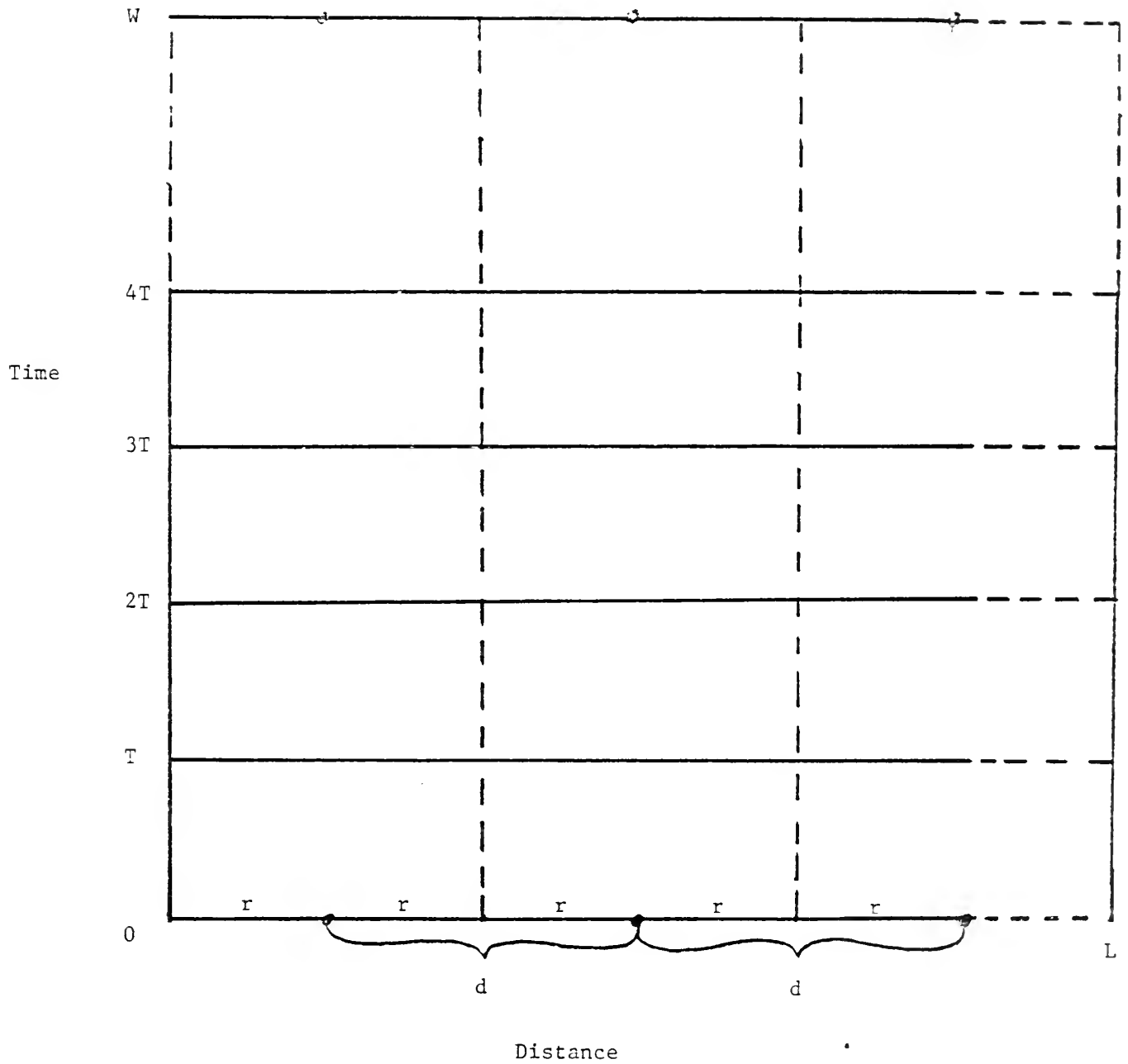
The firm's behavior and changes in that behavior involve complex interactions between spatial and temporal factors. Thus, a change in a spatial parameter, say the rate that carriers charge to transport the firm's product, has direct effects on such geographic choice variables as the spacing of plants and the size of the market region each plant serves. However, such a change also has cross effects on elements of the firm's temporal plan, including the amount produced at various points of time, the inventory carried, etc. Of course, changes in temporal parameters, such as the interest rate and the cost of storage, also have two sets of effects, direct effects on elements of the firm's temporal plan and cross effects on its spatial choices.

The remainder of the paper is divided into three parts. The model of the firm that maximizes profits by operating across time and space is presented in Part II. Part III is devoted to an investigation of the comparative statics-dynamics of the model. We examine the effects on the firm's behavior of changes in the costs of carrying goods through time and space. Some of the limitations of the model and their implications for future research appear in Part IV.

II. The Model

Consider a firm that can sell over a market region of length L , a time horizon of duration W , and which faces cost and demand functions that are time and space invariant. That is, for each space-time pair at which the firm can produce, it would incur identical costs if it produced identical outputs and it would sell identical quantities if it charged identical prices. Given this assumption and an additional simplifying assumption mentioned below, each plant that the firm constructs will service a market region of identical length. In addition, sales from each production-sales run lasts for the identical length of time. We call the constant interplant distance d . $\frac{d}{2}$, the distance from the plant to each of the two furthest markets served by that plant, is called the market radius and is denoted by r . The constant duration of production-sales runs is denoted by T , and since we rule out backordering (i.e., the satisfaction of today's consumption from future outputs) there is no temporal equivalent to radius. Figure 3.1 below depicts the spatial-temporal choices the firm faces. Distance and the spacing of plants are shown on the horizontal axis. Time is measured on the vertical axis. Each column of dots is associated with a plant location. We refer to the time between successive dots in a column as the length of a production-sales run. That is, each dot within a column denotes an instant of time at which production takes place. At that time, the stock of inventory from the preceding production time is exhausted. The dashed vertical lines and the two solid vertical lines denote plant market boundaries. Each plant services the market region contained within its left most and right most

Figure 1: Time-Distance Grid



T, Time between production-sales runs
W, Planning horizon
L, Market length
d, Distance between plants
r, Market radius

boundary. The solid horizontal lines denote temporal production-sales run boundaries. The output of a plant's production run is sold at all locations within the plant's market region until the time of the next production run. Production takes place when output of the previous run has been exhausted by sales.

Given such a grid specifying interplant spacing and the time between production-sales runs, the firm chooses the output of each run and the associated sales policy that maximize profit. Product is costly to ship through space, costly to store through time, and the firm discounts future revenue and cost flows. The output, sales policy, and profit associated with a single run at a given plant depends on freight costs, storage costs, and discount rates as well as demand and production cost function. We denote the maximal profit associated with a single plant production-sales run by $\pi(r,T)$.

If there are n plants each serving a market region of identical length, and each plant is located in the center of the region, then $d = \frac{L}{n}$ and $r = \frac{L}{2n}$. Alternately, $n = \frac{L}{2r}$. Since production-sales runs at each plant are completed at the identical time, to maximize aggregate profits of all plants from a single production-sales run, the firm maximizes $n\pi(r,T) = \frac{L}{2r} \pi(r,T)$. Since $\frac{L}{2}$ is fixed, maximization of $n\pi(r,T)$ is equivalent to maximization of $\frac{\pi(r,T)}{r}$. This definition of aggregate profit assumes n is an integer. When r is chosen so that $\frac{L}{2r}$ is not integer valued, it is not strictly correct to assume that all plants will produce identical outputs and consequently also incorrect to assume that they are all equally spaced. We want to avoid this integer problem caused by assuming that the endpoints of the entire

market region served by the firm are fixed. Consequently we assume that when the firm optimizes with regard to interplant spacing it maximizes $\frac{\pi(r,T)}{r}$, an assumption that has been employed by others interested in location and spatial competition.²

Since the firm discounts future revenue and cost flows at a constant discount rate equal to δ , the discounted present value of all production runs from a given plant equals:

$$(1) \quad \sum_{j=0}^{n-1} e^{-\delta jT} \pi(r,T) = \frac{1-e^{-\delta nT}}{1-e^{-\delta T}} \pi(r,T)$$

where $n = \frac{W}{T}$.³ The integer problem crops up here as well. However, if W is very large relative to T , this discounted present value is approximately equal to $\frac{1}{1-e^{-\delta T}} \pi(r,T)$. We take this to be the objective of the firm with regard to its choice of time between production-sales runs. This is the objective when the time horizon is infinite and consequently no endpoint problem arises.

It should be noted that the firm maximizes average profit per unit distance when making its interplant spacing choice but maximizes aggregate discounted profits in its choice of time between production runs. While there is much similarity between the two choices we will not obtain symmetric conditions for the optimal r and T . Note however, that when W is finite and $\delta = 0$ the two problems are essentially identical.

Consequently we model the firm's problem as follows:

$$(2) \quad \underset{r, T \geq 0}{\text{maximize}} \quad \frac{\pi(r,T)}{(1-e^{-\delta T})r}.$$

The first order conditions for an interior optimum of this problem are:

$$(3) \quad \pi_r - \frac{\pi(r,T)}{r} = 0$$

$$\text{and} \quad \pi_T - \frac{\delta e^{-\delta T} \pi(r,T)}{(1-e^{-\delta T})} = 0$$

The second order sufficient conditions require that:

$$(4) \quad \begin{bmatrix} \pi_{rr} & \pi_{rT} - \frac{\pi_T}{r} \\ \pi_{rT} - \frac{\pi_T}{r} & \pi_{TT} + \frac{\delta^2}{e^{\delta T} - 1} \pi \end{bmatrix} \quad \text{is negative definite.}$$

To study this problem more closely we examine the determinants of $\pi(r,T)$. Assume for now that r and T are fixed. We first look at the variational problem of how to sell over time and space when the output level from a production run, Q , is held fixed. The firm solves the following problem:

$$(5) \quad \begin{array}{l} \text{maximize } 2 \int_0^T \left\{ \int_0^r e^{-\delta t} [p(y(z,t)) - fz] y(z,t) dz (-sI(t)) \right\} dt \\ y(\cdot) \geq 0 \end{array}$$

$$\text{subject to: } I(t) = Q - 2 \int_0^t \int_0^r y(z,u) dz du$$

$$I(t) \geq 0 \quad \text{for } t \in [0, T].$$

(6) where z is an index of distance from the plant,
 t is an index of time from production,
 $y(z,t)$ is sales at (z,t) ,

f is the unit freight rate,

s is the unit storage cost,

$I(t)$ is inventory at t ,

δ is the discount rate, and

$p(y)$ is the sales price when sales are y .

We explain this problem as follows. All revenue flows at time t are discounted to time zero by the discount factor $e^{-\delta t}$. Gross revenue in current value from sales at (z,t) is $p(y(z,t))y(z,t)$. Each unit of product shipped from the plant to a market z units from the plant requires payment of a freight charge with unit price in current value equal to fz . Thus the total freight charge associated with sales at (z,t) in current value is $fzy(z,t)$. When we aggregate over all market locations within the plant's market region, the discounted revenue net of freight charges at time t is $2e^{-\delta t} \int_0^r [p(y(z,t)) - fz]y(z,t)dz$. For each unit of product held in inventory at time t the firm pays a storage charge with unit price s in current dollars. Given inventory level of $I(t)$, at time t the discounted value of storage charges is $e^{-\delta t} sI(t)$. Inventory at t equals the original quantity available for sale, Q , minus the amount sold up to t , $2 \int_0^t \int_0^r y(z,u)dzdu$. The requirement that inventory is nonnegative for all t in $[0,T)$ amounts to requiring that over this interval not more is sold in total than was available for sale, i.e., $2 \int_0^T \int_0^r y(z,t)dzdt \leq Q$.

It is convenient to assume that the last inequality holds as an equality so that all product available for sale is actually sold. Doing so allows us to treat inventory charges in terms of sales flow

rather than in terms of inventory stock, and makes them more directly analogous to freight charges. Since $I(u)$ can be rewritten as:

$$(7) \quad 2 \int_u^T \int_0^r y(z,t) dz dt = I(u)$$

the total discounted present value of storage charges is obtained in terms of sales via a change in the order of integration. Thus:

$$\begin{aligned} (8) \quad \int_0^T e^{-\delta u} s I(u) du &= \int_0^T e^{-\delta u} s \left[2 \int_u^T \int_0^r y(z,t) dz dt \right] du \\ &= 2 \int_0^T \int_0^r s y(z,t) \int_0^t e^{-\delta u} du dz dt \\ &= 2 \int_0^T \int_0^r \left(\frac{1-e^{-\delta t}}{\delta} \right) s y(z,t) dz dt \\ &= 2 \int_0^T \int_0^r e^{-\delta t} \left(\frac{e^{\delta t}-1}{\delta} \right) s y(z,t) dz dt. \end{aligned}$$

In current value, the unit storage cost associated with sales at time t is $\left(\frac{e^{\delta t}-1}{\delta} \right) s$. Given this way of writing inventory costs our problem can be rewritten as:

$$\begin{aligned} (9) \quad \max_{y(\cdot) \geq 0} \quad & 2 \int_0^T \int_0^r e^{-\delta t} \left[p(y(z,t)) - f z - \frac{s(e^{\delta t}-1)}{\delta} \right] y(z,t) dz dt \\ \text{subject to} \quad & 2 \int_0^T \int_0^r y(z,t) dz dt = Q. \end{aligned}$$

We call the value of the objective when evaluated along the optimal sales trajectory, $TR(r,T,Q)$.

Note that in formulating this problem the only restriction placed on sales at (z,t) is that they are nonnegative. Hence we allow for

both interspatial and intertemporal price discrimination. Besides being theoretically appealing, price discrimination has the additional advantage of allowing us to describe the optimal sales policy with intuitive first order conditions. Note however that if such discrimination is not possible, due to regulation or to fear of potential entry, the model can be reformulated to require uniform delivered prices.⁴

The first order conditions governing the optimal sales policy are:

$$(10) \quad e^{-\delta t} [p(y(z,t)) + p'(y(z,t))y(z,t) - fz - \frac{s(e^{\delta t} - 1)}{\delta}] \leq k$$

for all (z,t) ,

$$y(z,t) \geq 0 \quad \text{for all } (z,t), \text{ and}$$

$$y(z,t) \{k - e^{-\delta t} [p(y(z,t)) + p'(y(z,t))y(z,t) - fz - s(e^{\delta t} - 1)]\} = 0$$

for all (z,t) .

These first order conditions along with the condition $2 \int_0^T \int_0^r y(z,t) dz dt = Q$ determine the optimal sales policy.⁵ We have assumed that the spot demand curve and its corresponding marginal revenue curve are downward sloping. Hence the first order conditions imply that at each location, z , (gross) marginal revenue rises linearly in distance since unit freight charges have been assumed to rise linearly in distance. Therefore, sales fall with distance from the plant. Likewise at each time, t , current value (gross) marginal revenue rises over time as the current value of storage charges rise and

future net revenue flows are discounted more heavily than current net revenue flows. Hence sales fall from the instant of production to the instant at which sales exhaust the output of a production run.

Since spot marginal revenue curves are downward sloping we can conclude $\frac{\partial^2 TR}{\partial Q^2} < 0$. To determine the optimal output, the firm solves the following maximization problem.

$$(11) \quad \underset{Q \geq 0}{\text{maximize}} \quad TR(r, T, Q) - C(Q).$$

The first order conditions for this problem are:

$$(12) \quad \frac{\partial TR}{\partial Q} - C'(Q) = 0.$$

The second order conditions are:

$$(13) \quad \frac{\partial^2 TR}{\partial Q^2} - C''(Q) < 0.$$

We assume that $C'(Q) > 0$, $\frac{C(Q)}{Q}$ is U-shaped, and there are no fixed costs, i.e., $C(0) = 0$. This requires $C'(Q)$ to be U-shaped as well. Hence there will be two, one, or no solutions to the first order conditions. When there are none, the firm shuts down. When there is one solution the second order conditions will be satisfied, i.e., $\frac{\partial TR}{\partial Q}$ cuts $C'(Q)$ from above. When there are two solutions only one will satisfy the second order conditions. The other will actually be a local minimum of the objective function. At the smaller of the two output levels which satisfy the first order conditions, $\frac{\partial TR}{\partial Q}$ cuts $C'(Q)$ from below. At the larger it cuts from above. Hence the larger of the two output levels is the only candidate for an interior optimum.

We can now return to the choice of r and T . Let $y(z,t,r,T)$ denote optimal sales at location-time pair (z,t) , when the market radius is r and the time between production-sales runs is T . We drop the explicit functional dependence on r and T and write these sales as $y(z,t)$. Let $Q(r,T) = 2 \int_0^T \int_0^r y(z,t) dz dt$ be the optimal production, given r and T . Again we drop the explicit functional dependence on r and T and write this production as Q . We can now write:

$$(14) \quad \pi(r,T) = 2 \int_0^T \int_0^r e^{-\delta t} [p(y(z,t)) - fz - \frac{s(e^{\delta t} - 1)}{\delta}] y(z,t) dz dt - C(Q).$$

$$(15) \quad \pi_r = 2 \int_0^T e^{-\delta t} [p(y(r,t)) - fr - \frac{s(e^{\delta t} - 1)}{\delta}] y(r,t) dt \\ - C'(Q) \cdot 2 \int_0^T y(r,t) dt.$$

$\frac{\partial Q}{\partial r} = 2 \int_0^T y(r,t) dt$ and the terms involving $\frac{\partial y(z,t)}{\partial r}$ are not included above because, by the envelope theorem, this change has no effect on overall profits. Likewise

$$(16) \quad \pi_T = 2 \int_0^r e^{-\delta t} [p(y(z,T)) - fz - \frac{s(e^{\delta T} - 1)}{\delta}] y(z,T) dz \\ - C'(Q) \cdot 2 \int_0^r y(z,T) dz$$

$$\text{where } \frac{\partial Q}{\partial T} = 2 \int_0^r y(z,t) dt.$$

We can also obtain the second own and cross partials as follows:

$$(17) \quad \pi_{rr} = 2 \int_0^T e^{-\delta t} [-f y(r,T)] dt - C''(Q) \frac{\partial Q}{\partial r} \frac{dQ}{dr}$$

$$\text{where } \frac{dQ}{dr} = 2 \int_0^T \int_0^r \frac{\partial y(z,t)}{\partial r} dz dt + \int_0^T y(r,t) dt.$$

Note that $\frac{dQ}{dr} \neq \frac{\partial Q}{\partial r}$. $\frac{dQ}{dr}$ includes two effects. The first occurs because sales at each (z,t) pair change when r changes. The second occurs because the market area changes when r changes. $\frac{\partial Q}{\partial r}$ only includes this second effect.

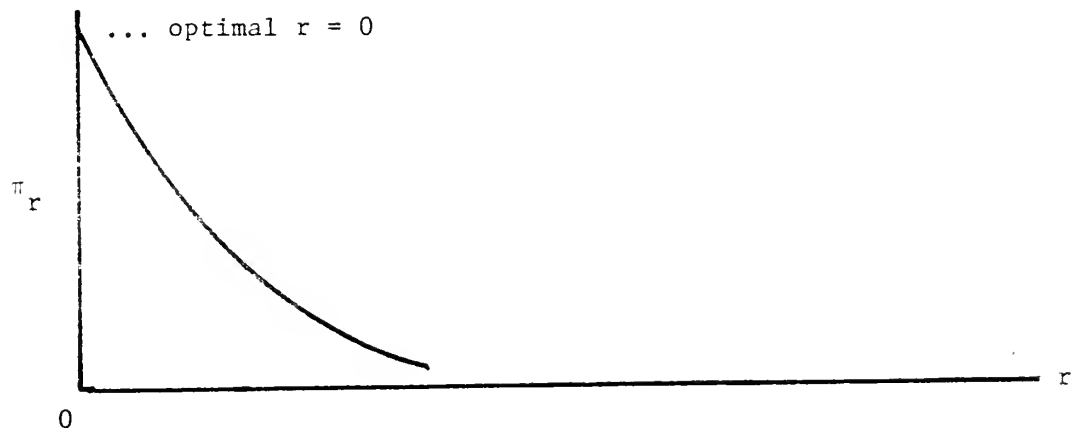
We presume that $\frac{dQ}{dr} > 0$ in the relevant range. Intuitively, raising r shifts the aggregate marginal revenue curve for the entire market region to the right. Since the marginal cost curve is unaffected optimal output must rise.

From the above we can conclude that $\pi_{rr} > 0$ only if $C'' < 0$. Since we are assuming that the marginal cost curve itself is U-shaped, one and only one of the situations depicted in Figure 2 can occur. The graphs in the Figure are constructed from the policy which solves the first-order conditions. Obviously, in this case these conditions do not ensure an optimum. We focus exclusively on an interior optimum (optimal $r > 0$). A necessary condition for an interior optimum is $\pi_{rr}|_{r=0} > 0$, while at the optimal point $\pi_{rr}|_{r=\text{optimum}} < 0$. Thus, second order conditions will not be satisfied universally and they must be checked before a comparative static-dynamic analysis is performed.

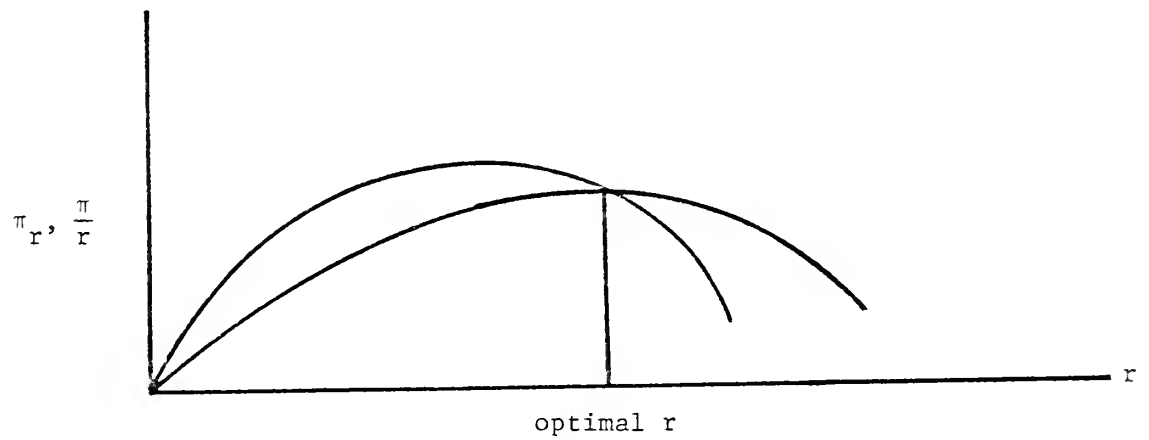
$$(18) \quad \pi_{rT} = 2e^{-\delta T} [p(y(r,T)) - fr - \frac{s(e^{\delta T} - 1)}{\delta}] y(r,T) - C''(Q) \frac{\partial Q}{\partial r} \frac{dQ}{dT} \\ - C'(Q) \frac{\partial^2 Q}{\partial r \partial T}$$

Figure 2

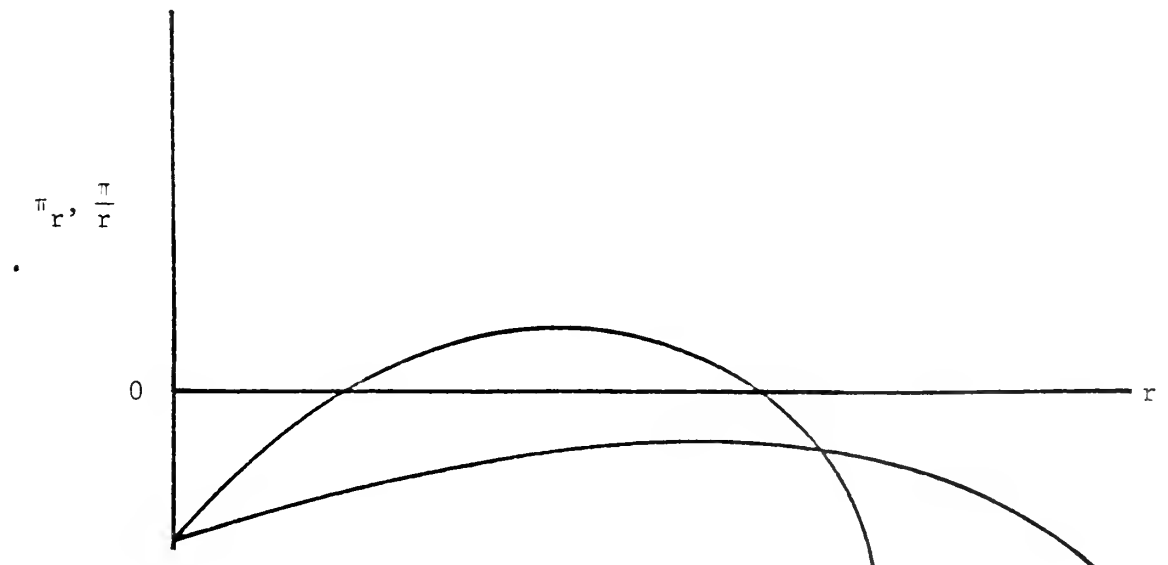
a) Plant at every point



b) Interior solution for interplant spacing



c) Firm does not operate



where $\frac{\partial^2 Q}{\partial r \partial T} = 2y(r, T)$.

Finally,

$$(19) \quad \pi_{TT} = 2 \int_0^r [-\delta e^{-\delta T} [p(y(z, T)) - fz - \frac{s(e^{\delta T} - 1)}{\delta}] y(z, T) - sy(z, T)] dz \\ - C''(Q) \frac{\partial Q}{\partial T} \frac{dQ}{dT}.$$

In order to perform the comparative statics-dynamics we must be able to sign $\pi_{rT} - \frac{\pi_T}{r}$.

$$(20) \quad \pi_{rT} - \frac{\pi_T}{r} = 2 \{ e^{-\delta T} [p(y(r, T)) - fr - \frac{s(e^{\delta T} - 1)}{\delta}] - C'(Q) \} y(r, T) \\ - C''(Q) \frac{\partial Q}{\partial r} \frac{dQ}{dT} = 2 \int_0^r \frac{\{ e^{-\delta T} [p(y(z, T)) - fz - \frac{s(e^{\delta T} - 1)}{\delta}] - C'(Q) \} y(z, T) dz}{r}$$

Discounted revenue net of freight, storage, and marginal production cost (undiscounted) must be falling with distance from the plant, z . This follows since: (a) by the first order conditions, sales fall with z ; and (b) if discounted net revenue were to rise with z over some interval, $[z_1, z_2]$, then by setting sales over this entire interval equal to sales at z_2 the firm would increase profits. Profits would rise because unit freight charges over $[z_1, z_2]$ are greatest at z_2 so that net revenue at z would be at least as great as net revenue at z_2 , while production costs would fall since a cutback in sales would imply a cutback in production as well.

We conclude that if $C''(Q) \geq 0$ certainly $\pi_{rT} - \frac{\pi_T}{r} < 0$. When $C''(Q) < 0$, $\pi_{rT} - \frac{\pi_T}{r}$ may change signs. This completes the presentation of the model.

III. Comparative Statics-Dynamics

In this section we analyze how the optimal sales, production, inter-plant spacing, and duration of production-sales runs are affected by a change in freight costs, storage costs, or the discount rate. To perform this analysis we view $y(z,t)$, Q , r , and T as functions of the parameters f , s , and δ . By totally differentiating the two equation system given in (3) with respect to any one of these parameters we can obtain expressions for the derivatives of optimal policy variables with respect to the parameter change. In general these expressions are difficult to sign. In order to disentangle the complex effects they entail we present several preliminary cases where the analysis is made easier by assuming that either T or r is fixed. In addition, this procedure allows us to compare the spatial problem with its temporal analog.

Case 1: The Pure Spatial Model

This case focuses on pure spatial adjustments under the assumptions that T is fixed and $\delta = s = 0$. Our first order conditions then imply $y(z,t) = y(z,0)$ for all t . Hence

$$\begin{aligned} (21) \quad \pi(r,T) &= 2 \int_0^T \int_0^r [p(y(z,t)) - fz] y(z,t) dz dt - C(Q) \\ &= 2T \int_0^r [p(y(z,0)) - fz] y(z,0) dz - C(Q) \end{aligned}$$

$$\text{where } Q = 2T \int_0^r y(z,0) dz.$$

The firm maximizes $\frac{\pi(r,T)}{r}$ with respect to r . The relevant first order condition is:

$$(22) \quad 2T[p(y(r,0)) - fr - C'(Q)]y(r,0) - \frac{2T \int_0^r [p(y(z,0)) - fz] y(z,0) dz + C(Q)}{r} = 0.$$

The relevant second order condition is:

$$(23) \quad -2Tfy(r,0) - 2TC''(Q)y(r,0) \frac{dQ}{dr} < 0.$$

Thus $f > C''(Q) \frac{dQ}{dr}$ is required. At an interior optimum we obtain the following comparative static result.

$$(24) \quad \text{sign } \frac{\partial r}{\partial f} = \text{sign} \left[T \left[-r - C''(Q) \frac{dQ}{df} \right] y(r,0) - \frac{T \int_0^r -zy(z,0)dz}{r} \right] \\ = \text{sign} \left[\left[-r - C''(Q) \frac{dQ}{df} \right] y(r,0) + \frac{\int_0^r zy(z,0)dz}{r} \right].$$

Intuitively, $\frac{\partial r}{\partial f}$ should be negative. Raising freight rates raises unit delivery costs more the further one gets from the plant since unit delivery costs at z are fz . An increase in the freight rate, f , makes markets further from the plant relatively less attractive than markets closer to the plant. Hence it pays for the firm to space plants closer together to avoid these high delivery costs, though this strategy cuts down on the firm's ability to exploit plant scale economies and consequently leads to higher average production costs. A sufficient condition for this intuition to hold is that

$ry(r,0) > \frac{\int_0^r zy(z,0)dz}{r}$ and that $C''(Q) \leq 0$. We will actually assume a stronger condition on the sales path, namely, that $zy(z,0)$ is increasing in z . This condition says that total outlays on freight increase with distance from the plant. When this condition holds we say that sales are spatially elastic.

Perverse results concerning the sign of $\frac{\partial r}{\partial f}$, i.e., $\frac{\partial r}{\partial f} > 0$ can occur

if $ry(r,0) < \frac{\int_0^r zy(z,0)dz}{r}$ or if $C''(Q) > 0$. If the former holds then an increase in the freight rate hurts the firm relatively less at the boundary of the market region than it does on average over the market region. If the latter holds, raising f for a fixed market region lowers marginal production cost since output falls. Though gross marginal revenue at the boundary of the market region rises when the freight rate rises, net marginal revenue actually falls. This is the case because by the first order conditions, marginal production cost must be equal to net marginal revenue. In these circumstances, it is possible for revenue earned on sales at the boundary, net of delivery and marginal production costs, to fall off less with an increase in the freight rate than the net revenue lost on average over the entire market region. If this is the case, it actually pays for the firm to increase the spacing between plants, i.e., the market region increases with an increase in the freight rate.

Case 2

This case is the temporal analog of Case 1. It considers pure temporal adjustments to changes in the cost of storage when the freight rate is set equal to zero and r is fixed. In this case we will have $y(z,t) = y(0,t)$ for all z . Hence

$$\begin{aligned}
 (25) \quad \pi(r,T) &= 2 \int_0^T \int_0^r e^{-\delta t} [p(y(z,t)) - \frac{s(e^{\delta t}-1)}{\delta}] y(z,t) dz dt - C(Q) \\
 &= 2r \int_0^T e^{-\delta t} [p(y(0,t)) - \frac{s(e^{\delta t}-1)}{\delta}] y(0,t) dt - C(Q)
 \end{aligned}$$

where $Q = 2r \int_0^T y(0, \tau) d\tau$.

The firm maximizes $\frac{1}{1-e^{-\delta T}} \pi(\tau, T)$ with respect to T . The relevant first order condition is:

$$(26) \quad 2r[e^{-\delta T} [p(y(0, T)) - \frac{s(e^{\delta T} - 1)}{\delta}] - C'(Q)] y(0, T) \\ - \frac{\delta e^{-\delta T}}{1-e^{-\delta T}} [2r \int_0^T e^{-\delta \tau} [p(y(0, \tau)) - \frac{s(e^{\delta \tau} - 1)}{\delta}] y(0, \tau) d\tau - C(Q)] = 0.$$

The relevant second order condition is:

$$(27) \quad 2r[-\delta e^{-\delta T} [p(y(0, T)) - \frac{s(e^{\delta T} - 1)}{\delta}] - s - C''(Q) \frac{dQ}{dT}] y(0, T) \\ + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} [2r \int_0^T [p(y(0, \tau)) - \frac{s(e^{\delta \tau} - 1)}{\delta}] y(0, \tau) d\tau - C(Q)] < 0.$$

This condition is a bit more complicated than the equivalent condition of the previous case since now we are maximizing aggregate discounted present value profits rather than profits per unit length. At an interior optimum we obtain the following result:

$$(28) \quad \text{sign } \frac{\partial T}{\partial s} = \text{sign} [2r[e^{-\delta T} (\frac{e^{\delta T} - 1}{\delta}) - C''(Q) \frac{dQ}{ds}] y(0, T) \\ + \frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \cdot 2r \int_0^T e^{-\delta \tau} (\frac{e^{\delta \tau} - 1}{\delta}) y(0, \tau) d\tau] \\ = \text{sign} [e^{-\delta T} (\frac{e^{\delta T} - 1}{\delta}) - C''(Q) \frac{dQ}{ds}] y(0, T) \\ + \frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \int_0^T e^{-\delta \tau} (\frac{e^{\delta \tau} - 1}{\delta}) y(0, \tau) d\tau.]$$

As in the spatial case one would expect intuitively that $\frac{\partial T}{\partial s} < 0$. A sufficient condition for this result to hold is that $C''(Q) \leq 0$ and $\frac{(e^{\delta t}-1)}{\delta} y(0,t)$ is increasing with t . That is, current value storage charges associated with sales at t when the interest rate is δ are increasing with time from production. This is the temporal analog of the spatial elasticity assumption. We note that $\lim_{\delta \rightarrow 0} \frac{e^{\delta t}-1}{\delta} = t$ so that these conditions coincide when there is no discounting. When this condition holds $\frac{(e^{\delta t}-1)}{\delta} y(0,t) < \frac{(e^{\delta T}-1)}{\delta} y(0,T)$ and consequently

$$\begin{aligned} \frac{\delta e^{-\delta T}}{(1-e^{-\delta T})} [2r \int_0^T e^{-\delta t} \frac{(e^{\delta t}-1)}{\delta} y(0,t) dt] &< \\ \frac{\delta e^{-\delta T}}{(1-e^{-\delta T})} [2r \int_0^T e^{-\delta t} \frac{(e^{\delta T}-1)}{\delta} y(0,T) dt] & \\ = e^{-\delta T} [2r \frac{(e^{\delta T}-1)}{\delta} y(0,T)] & . \end{aligned}$$

Perhaps a more revealing way to compare the temporal and spatial analyses of Cases 1 and 2 is to rewrite the first order conditions as follows:

For the spatial analysis,

$$(29) \quad \frac{\partial \pi}{\partial r} - \frac{\pi}{r} = 0.$$

For the temporal analysis,

$$(30) \quad \frac{\partial \pi}{\partial T} - \frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi = \frac{\partial \pi}{\partial T} - \frac{\delta T}{e^{\delta T}-1} \frac{\pi}{T} = 0.$$

In the temporal case, marginal profit per length of time between production-sales runs equals a fraction, $\frac{\delta T}{e^{\delta T} - 1}$, of average profit. Since

$\lim_{\delta T \rightarrow 0} \frac{\delta T}{e^{\delta T} - 1} = 1$ when $\delta = 0$, we get $\frac{\partial \pi}{\partial T} - \frac{\pi}{T} = 0$. In this case the spatial

and temporal models coincide. Since $\lim_{\delta T \rightarrow \infty} \frac{\delta T}{e^{\delta T} - 1} = 0$ when δ is very large, we get approximately $\frac{\partial \pi}{\partial T} = 0$. That is, the firm maximizes profit by acting as if there is only one production run when δ is large. We now consider the comparative dynamics of changes in δ .

If the marginal revenue curve has a finite valued y intercept (e.g., linear demand) then $\frac{\partial \pi}{\partial T} = 0$ for some finite T . Furthermore, the T where this occurs is decreasing in δ . Intuitively one has the following:

if $\frac{\partial \pi}{\partial T} \gg 0$ then $\frac{\partial T}{\partial \delta} > 0$. That is, raising δ increases the relative importance of profits from the first production-sales run. When $\frac{\partial \pi}{\partial T} \gg 0$ these profits can be increased significantly by increasing T . When $\frac{\partial \pi}{\partial T} \approx 0$, changing T has no significant effect on profits from the first production run. In this case $\frac{\partial T}{\partial \delta} < 0$.

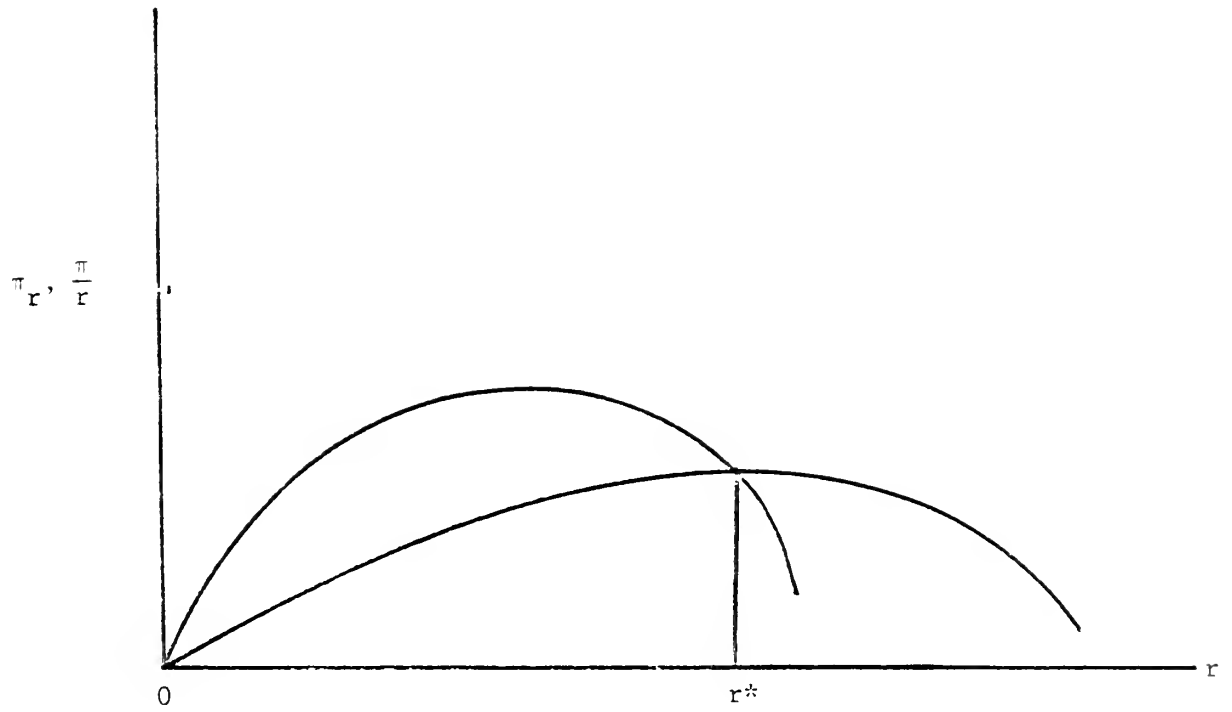
Alternatively, one can view these effects in terms of Figure 3. Raising δ shifts down both $\frac{\partial \pi}{\partial T}$ and $\frac{\pi}{T}$. It also lowers the fraction $\frac{\delta T}{e^{\delta T} - 1}$. It is reasonable that $\frac{\partial \pi}{\partial T}$ shifts down more than $\frac{\pi}{T}$. Thus, isolating this shifting effect, T has a tendency to fall. On the other hand reducing $\frac{\delta T}{e^{\delta T} - 1}$ has a tendency to increase T .

Case 3: T Held Fixed

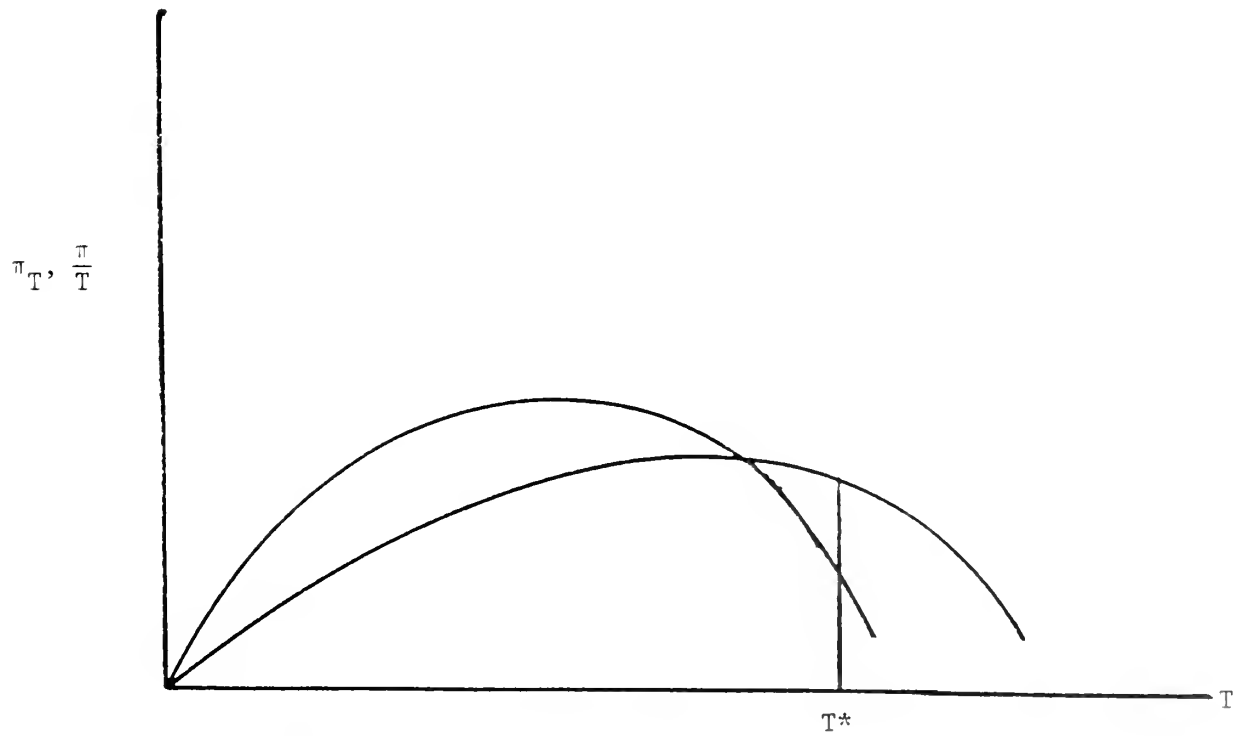
Here we return to an analysis of the effects of changes in f , the freight rate. Again T , the length of time of a production-sales run, is held constant. However, this case differs from Case 1 in that s and δ are positive.

Figure 3

a) Spatial



b) Temporal



* Indicates optimal value

Now, the relevant first order condition is:

$$(31) \quad \frac{2 \int_0^T e^{-\delta t} [p(y(r,t) - fr - s \frac{(e^{\delta t} - 1)}{\delta})] y(r,t) dt - C'(Q) \cdot 2 \int_0^T y(r,t) dt}{2 \int_0^T \int_0^r e^{-\delta t} [p(y(z,t) - fz - s \frac{(e^{\delta t} - 1)}{\delta})] y(z,t) dz dt - C(Q)} = 0.$$

The relevant second order condition is:

$$(32) \quad 2 \int_0^T e^{-\delta t} [-fy(r,t)] dt - C''(Q) \frac{dQ}{dr} \cdot 2 \int_0^T y(r,t) dt < 0.$$

At an interior optimum the following results are obtained:

$$(33) \quad \text{sign } \frac{\partial r}{\partial f} = \text{sign} \left[2 \int_0^T [-e^{-\delta t} r - C''(Q) \frac{dQ}{df}] y(r,t) dt + \frac{2 \int_0^T \int_0^r e^{-\delta t} z y(z,t) dz dt}{r} \right].$$

This is quite similar to the results reported above for the analysis of the pure spatial effects of changes in f . If $zy(z,t)$ is increasing in z for each t and $C''(Q) \leq 0$ then $\frac{\partial r}{\partial f} < 0$. In the present case the market region is a rectangle. The spatial boundary of the market region is the right and left edges of that rectangle, i.e., all markets at a distance r from the plant over the interval $[0, T)$. When sales are spatially elastic at time t , an increase in the freight rate has relatively greater effect at the boundary than on average over the plant's entire market region at time t . When this is true for all t it is obviously also true in the temporal aggregate.

$$(34) \quad \text{sign } \frac{\partial r}{\partial s} = \text{sign} \left[2 \int_0^T \left[-\frac{(1-e^{-\delta t})}{\delta} - C''(Q) \frac{dQ}{ds} \right] y(r, t) dt \right. \\ \left. + \frac{2 \int_0^T \int_0^r \frac{(1-e^{-\delta t})}{\delta} y(z, t) dz dt}{r} \right].$$

Intuitively $\frac{dr}{ds} > 0$. As long as $C''(Q) \geq 0$ this is necessarily the case because

$$\frac{\int_0^r \left(\frac{1-e^{-\delta t}}{\delta} \right) y(z, t) dz}{r} > \left(\frac{1-e^{-\delta t}}{\delta} \right) y(r, t)$$

since sales fall with distance from the plant. Since sales are decreasing with distance from the plant an increase in s raises storage cost more near the plant than away from it. Cost rises least of all at boundary markets. Given this relative cost differential, the optimal radius should increase. In addition, increasing s can be thought of as shifting the aggregate marginal revenue curve to the left. Thus the intersection of aggregate marginal revenue and marginal cost changes with s . When $C''(Q) > 0$ this effect also tends to increase the optimal radius.

We also can sign $\frac{\partial r}{\partial \delta}$!

$$(35) \quad \text{sign } \frac{\partial r}{\partial \delta} = \text{sign} \left[2 \int_0^T \left\{ -te^{-\delta t} \left[p(y(r, t)) - fr - s \left(\frac{e^{\delta t} - \delta t - 1}{t\delta^2} \right) \right] \right. \right. \\ \left. \left. - C''(Q) \frac{dQ}{d\delta} \right\} y(r, t) dt \right. \\ \left. + \frac{2 \int_0^T \int_0^r te^{-\delta t} \left[p(y(z, t)) - fz - s \left(\frac{e^{\delta t} - \delta t - 1}{t\delta^2} \right) \right] y(z, t) dz dt}{r} \right]$$

Note that $[p(y(z,t)) - fz - s(\frac{s^{\delta t} - \delta t - 1}{t\delta^2})]y(z,t) \geq [p(y(z,t)) - fz - s(\frac{e^{\delta t} - 1}{\delta})]y(z,t)$.

(The second expression is net delivered total revenue and is declining in z .) Since $y(z,t)$ is declining in z the first expression is also declining in z . Hence $\frac{dr}{d\delta} > 0$ as long as $C''(Q) \geq 0$.

Case 4: r Held Fixed

This is the case that is the temporal analog of Case 3. Now, r is fixed and T is variable. Again, f and δ are positive.

The relevant, first order condition for this case is:

$$(36) \quad 2 \int_0^r \{e^{-\delta T} [p(y(z,T)) - fz - s(\frac{e^{\delta T} - 1}{\delta})] - C'(Q)\} y(z,T) dz \\ - \frac{\delta e^{-\delta T}}{1 - e^{-\delta T}} \{2 \int_0^T \int_0^r e^{-\delta t} [p(y(z,t)) - fz - s(\frac{e^{\delta t} - 1}{\delta})] dz dt - C(Q)\} = 0.$$

The relevant second order condition is:

$$(37) \quad 2 \int_0^r \{-\delta e^{-\delta T} [p(y(z,T)) - fz - s(\frac{e^{\delta T} - 1}{\delta})] - sC''(Q)\frac{dQ}{dT}\} y(z,T) dz \\ + \frac{\delta^2 e^{-\delta T}}{1 - e^{-\delta T}} \{2 \int_0^T \int_0^r e^{-\delta t} [p(y(z,t)) - fz - s(\frac{e^{\delta t} - 1}{\delta})] dz dt - C(Q)\} < 0.$$

At an interior optimum the following results are obtained.

$$(38) \quad \text{sign } \frac{\partial T}{\partial s} = \text{sign} [2 \int_0^r \{-(\frac{1 - e^{-\delta T}}{\delta}) - C''(Q)\frac{dQ}{ds}\} y(z,T) dz \\ - \frac{\delta e^{-\delta T}}{1 - e^{-\delta T}} \cdot 2 \int_0^T \int_0^r (\frac{1 - e^{-\delta t}}{\delta}) y(z,t) dz dt].$$

This is also similar to Case 2. If $\frac{e^{\delta t}-1}{\delta} y(z,t)$ is increasing in t for each z then $\frac{\partial T}{\partial s} < 0$ as long as $C''(Q) \leq 0$.

$$(39) \quad \text{sign } \frac{\partial T}{\partial f} = \text{sign} \left[2 \int_0^r \left\{ -ze^{-\delta T} - C''(Q) \frac{dQ}{df} \right\} y(z,T) dz \right. \\ \left. - \frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \cdot 2 \int_0^T \int_0^r -e^{-\delta t} z y(z,t) dz dt \right].$$

$\frac{\partial T}{\partial f} > 0$ as long as $C''(Q) \geq 0$.

Note that in cases 3 and 4 the cross effects, i.e., $\frac{\partial r}{\partial s}$ in case 3 and $\frac{\partial T}{\partial f}$ in case 4 are positive as long as $C''(Q) > 0$ independent of any condition on sales. This completes the short run analysis.

Case 5: The Long Run

In the long we have to take account that both r and T may vary.

Recall our second order conditions are:

$$(40) \quad \begin{bmatrix} \pi_{rr} & \pi_{rT} - \frac{\pi_T}{r} \\ \pi_{rT} - \frac{\pi_T}{r} & \pi_{TT} + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} \pi \end{bmatrix} \quad \text{is negative definite.}$$

This requires $\pi_{rr}, \pi_{TT} + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} \pi < 0$ and $\pi_{rr} \left[\pi_{TT} + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} \pi \right] - \left[\pi_{rT} - \frac{\pi_T}{r} \right]^2 > 0$

at an interior optimum.

Algebraically all the comparative static-dynamics results are given below.

$$(41) \quad \text{sign } \frac{\partial r}{\partial f} = \text{sign} \left[\left(\frac{\pi_f}{r} - \pi_{rf} \right) \left(\pi_{TT} + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} \pi \right) - \left(\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_f - \pi_{Tf} \right) \left(\pi_{rT} - \frac{\pi_T}{r} \right) \right].$$

$$(42) \quad \text{sign } \frac{\partial T}{\partial f} = \text{sign} \left[\left(\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_f - \pi_{Tf} \right) \pi_{rr} - \left(\frac{\pi_f}{r} - \pi_{rf} \right) \left(\pi_{rT} - \frac{\pi_T}{r} \right) \right].$$

$$(43) \quad \text{sign } \frac{\partial r}{\partial s} = \text{sign} \left[\left(\frac{\pi_s}{r} - \pi_{rs} \right) \left(\pi_{TT} + \frac{\delta^2 e^{-\delta T}}{1-e^{-\delta T}} \pi \right) - \left(\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_s - \pi_{Ts} \right) \left(\pi_{rT} - \frac{\pi_T}{r} \right) \right].$$

$$(44) \quad \text{sign } \frac{\partial T}{\partial s} = \text{sign} \left[\left(\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_s - \pi_{Ts} \right) \pi_{rr} - \left(\frac{\pi_s}{r} - \pi_{rs} \right) \left(\pi_{rT} - \frac{\pi_T}{r} \right) \right].$$

The interest rate effects are too complicated to include here. We have taken the liberty of putting in the "usual" signs of all terms above the corresponding term. When this is the case $\frac{\partial r}{\partial f}, \frac{\partial T}{\partial s} < 0$ and $\frac{\partial r}{\partial s}, \frac{\partial T}{\partial f} > 0$. We continue to assume the condition on sales that total freight outlays at distance z rise with z , and total current value storage charges at time t rise with t .

The "usual" signs may alter when $C'' \gg 0$ (to the right of min MC) or $C'' \ll 0$ (to the left of min MC).

When $C'' \gg 0$ the only terms which may switch signs are $\frac{\pi_f}{r} - \pi_{rf}$ which may become negative and $\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_s - \pi_{Ts}$ which may also become negative. When this happens $\frac{\partial r}{\partial f}, \frac{\partial T}{\partial s} > 0$ is possible as is $\frac{\partial r}{\partial s}, \frac{\partial T}{\partial f} < 0$.

When $C'' \ll 0$, $\pi_{rT} - \frac{\pi_T}{r} > 0$ is possible as is $\frac{\delta e^{-\delta T}}{1-e^{-\delta T}} \pi_f - \pi_{Tf}$,

$\frac{\pi_s}{r} - \pi_{rs} > 0$. When this is the case it is possible that

$$\frac{\partial r}{\partial f}, \frac{\partial T}{\partial f}, \frac{\partial r}{\partial s}, \frac{\partial T}{\partial s} < 0.$$

IV. Conclusion

In this paper we have presented a model of a monopolistic firm whose profit maximizing plan involves the operation of multiple plants that are dispersed in space, each plant with a distinct geographic market region. Within each such region, price rises and sales fall as distance from the plant increases because of transport costs. If the firm is free to vary delivered price, it will practice price discrimination even though it faces an identical demand function everywhere. The form of the discrimination is partial freight absorption.

Our firm's profit maximizing strategy also has temporal elements. It behaves dynamically even though the demand, production, and storage cost functions it faces over time are identical and unchanging, and the interest rate is also unchanging. The firm produces in excess of the quantity it sells during certain periods of time and uses its inventory to satisfy future demands. Each production-sales run constitutes a distinct temporal market. Each is the analog of a single plant's geographic market except that shipments only go in one direction.

Price rises and sales fall over each production-sales run because of inventory carrying costs. If the firm is free to vary price it will practice temporal price discrimination.

One of the goals of the research involved in this paper is to build a bridge between three rich bodies of literature in the theory of the firm; the theories of location, of dynamics, and of the multi-plant firm. Clearly, much remains to be done in constructing temporal-spatial models of the imperfectly competitive firm. Some of the most important aspects of the above bodies of literature are absent from our model. A few examples of kinds of things that would be very challenging and valuable to introduce into spatial-temporal models of the firm are presented below.

Consider the issue of differences in costs of production. Differences in such costs over time are a central part of the logic of many production-inventory models. Indeed, because they ignore economies of scale in production and assume fixed demands at each point in time, these models must assume that costs or demands vary with time in order to have a basis for dynamic behavior. The approach we have developed does not lend itself readily to the introduction of temporal differences in costs, unless certain spatial elements of the firm's profit maximizing plan are held fixed, i.e., the number and distribution of plants must be held constant.

If costs vary temporally the time between production runs will still change even if the above spatial elements are held constant. This means that our procedure of examining the typical production-sales run is no longer valid. In these circumstances one must determine the

optimal starting and ending time of each run and the optimum number of runs, the latter an integer problem.

Changes in costs of production over time mean changes in output. If outputs are to be produced in an optimal way, it may be necessary to alter the scale of plant associated with each production-sales run. However, suppose there are costs entailed in such alterations. We now have the problem of determining the optimum number of times that scale of plant should be changed. The introduction of such additional discreteness further limits our approach.

The above example dealt with the difficulties that arise because of differences in costs over time. Spatial variations in production costs entail difficulties of a similar nature. Thus, an important element in location theory is transport cost on raw and other material inputs that are only available at certain sites. Such transfer costs cause the firm's costs of production to vary over space. They enter into the determination of the optimum number and location of plants in some of the earliest location models. If the costs of shipping raw materials and finished products are significant each plant will have a different size market region. This is exactly the same problem discussed above for the temporal case. Let us consider one more example of a topic that it would be very valuable to introduce into our model.

Much of location theory deals with spatial competition between firms and groups of firms, but we have not taken up the question of how our firm should alter its temporal-spatial strategy if it believes that entry is possible. The problems that arise with entry are a good deal more interesting in the context of our model than in the usual analysis

of foreclosure. Our approach allows firms to adopt different strategies, some that give relatively more emphasis to spatial, and others that give relatively more emphasis to temporal elements.

Footnotes

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¹Below a nonexhaustive list of references is presented. Starred items contain extensive lists of additional references.

Multiplant Theory:

Patinkin (1947), Ginsberg (1974), (*)Scherer (1975), and Katz (1980).

Location and Spatial Competition Theory:

Fetter (1924), Predohl (1928), Hotelling (1929), Weber (1929), Palander (1935), Hoover (1937), Samuelson (1952), Losch (1954), (*)Isard (1956), Lefebvre (1958), Moses (1958), (*)Greenhut (1963), Alonso (1964), Christaller (1966), Von Thunen (1966), Beckman (1968), (*)Isard (1975), Holohan (1978), Philips and Thisse (1983).

Production and Inventory Theory:

Arrow, Karlin, and Scarf (1958), (*)Holt, Modigliani, Muth and Simon (1960), Mills (1962), Hadley and Whitin (1963), Whitin (1968), Peterson and Silver (1979), and Arvan and Moses (1982).

²For example see Katz (1980).

³When W is small it is inappropriate to assume that each production-sales run is identical in duration.

⁴Such a reformulation is presented in Arvan and Moses (1982).

⁵When $k > 0$ it is correct to assume that the initial stock, Q , is exhausted.

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